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MULTIMEDIA UNIVERSITY

FINAL EXAMINATION

TRIMESTER 3, 2018/2019

EMT1026 – ENGINEERING MATHEMATICS 2 (All Sections / Groups)

27th MAY 2019

9.00 AM – 11.00 AM

(2 Hours)

INSTRUCTIONS TO STUDENT:

1. This exam paper consists of **10 pages** (including cover page) with **4 Questions** only.
2. Attempt **all** 4 questions. All questions carry equal marks and the distribution of marks for each question is given.
3. Please print all your answers in the Answer Booklet provided. Show all relevant steps to obtain maximum marks.
4. Several tables are provided in the **Appendix** for your reference.

Question 1

- (a) Find the general solution to the following differential equation:

$$y'' - 3y' - 4y = xe^{4x}.$$

[12 marks]

- (b) Using the **power series** method, find the first **five nonzero** terms of the solution for the following differential equation:

$$(x-4)y' + y = 0.$$

[13 marks]

Question 2

The temperature distribution $u(x, t)$ along a uniform bar is described by the following initial-boundary value problem:

$$\frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial t},$$

$$u(0, t) = u(a, t) = 0, \quad t \geq 0,$$

$$u(x, 0) = \sin\left(\frac{\pi x}{a}\right), \quad 0 \leq x \leq a.$$

Using the method of separation of variables, find $u(x, t)$.

[25 marks]

Continued...

Question 3

(Note: The tables in APPENDIX B and APPENDIX C may be useful for solving this question.)

(a) Let $f(t) = t \sinh at$.

(i) Determine the Laplace transform of $f(t)$. [4 marks]

(ii) Rework part (i), but using a *different* transform property this time. Show that your final answer is still the same as that in part (i). [4 marks]

[Hint: $\sinh at = \frac{1}{2}(e^{at} - e^{-at})$.]

(b) Find the inverse Laplace transforms of the following s -domain functions:

(i) $\frac{1}{s(s-3)}$ [4 marks]

(ii) $\frac{1}{(s-3)^4}$ [4 marks]

(c) Consider the following ordinary differential equation:

$$y''(t) - 8y'(t) + 16y(t) = 4e^{5t}, \text{ subject to } y(0) = 1 \text{ and } y'(0) = 8.$$

By applying Laplace transform, show that

$$L\{y(t)\} = Y(s) = \frac{s-1}{(s-5)(s-4)}.$$

Hence, obtain the complete solution of the above differential equation.

[9 marks]

Continued...

Question 4

(Note: The tables in APPENDIX D, APPENDIX E and APPENDIX F may be useful for solving this question.)

- (a) Bags of cement that are packed by a particular machine have weights which are normally distributed with mean 50 kg and standard deviation 2 kg. Bags weighing less than 45 kg are rejected for being underweight, and those exceeding b kg are rejected for being overweight.

(i) Estimate the proportion of cement bags that are underweight. [4 marks]

(ii) Determine the value of b if 99% of the bags have acceptable weights. [4 marks]

- (b) A customer service center receives calls at an average rate of 1 call every 10 minutes.

(i) Compute the probability that more than 2 calls are received in the next 10 minutes. [4 marks]

(ii) If the operators of the service center break for lunch from 12.30p.m. – 1.00p.m., what is the probability that they do not miss any call? [4 marks]

- (c) A continuous random variable, X , has probability density function

$$f(x) = \begin{cases} \frac{1}{18}(x+3), & -3 \leq x \leq 3, \\ 0, & \text{otherwise.} \end{cases}$$

(i) Find the mean and variance of X . [5 marks]

(ii) Given that $E[aX + b] = 0$ and $Var(aX + b) = \frac{1}{2}$, determine the values of the constants a and b . [4 marks]

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APPENDIX A:**Half-range sine series:**

$$f(x) = \sum_{n=1}^{\infty} b_n \sin\left(\frac{n\pi x}{L}\right), \quad \text{where } b_n = \frac{2}{L} \int_0^L f(x) \sin\left(\frac{n\pi x}{L}\right) dx$$

Half-range cosine series:

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\left(\frac{n\pi x}{L}\right), \quad \text{where } a_0 = \frac{2}{L} \int_0^L f(x) dx, \quad a_n = \frac{2}{L} \int_0^L f(x) \cos\left(\frac{n\pi x}{L}\right) dx$$

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APPENDIX B:
Common Laplace Transform Pairs

	$f(t)$	$F(s) = L\{f(t)\}$
1.	1	$\frac{1}{s}$
2.	t	$\frac{1}{s^2}$
3.	t^n	$\frac{n!}{s^{n+1}}, n = 1, 2, \dots$
4.	e^{at}	$\frac{1}{s - a}$
5.	$\cos at$	$\frac{s}{s^2 + a^2}$
6.	$\sin at$	$\frac{a}{s^2 + a^2}$
7.	$\cosh at$	$\frac{s}{s^2 - a^2}$
8.	$\sinh at$	$\frac{a}{s^2 - a^2}$
9.	$u(t - a)$	$\frac{e^{-as}}{s}$
10.	$\delta(t - a)$	e^{-as}

Continued...

APPENDIX C: Laplace Transform Properties

	Property Name	Formula
1.	Linearity	$L\{af(t) + bg(t)\} = aL\{f(t)\} + bL\{g(t)\}$
2.	s - shifting	$L\{e^{at} f(t)\} = F(s - a)$
3.	Transform of Derivative	$L\{f'(t)\} = sL\{f(t)\} - f(0)$ $L\{f''(t)\} = s^2 L\{f(t)\} - sf(0) - f'(0)$
4.	Transform of Integration	$L\left\{\int_0^t f(\tau) d\tau\right\} = \frac{1}{s} F(s)$
5.	t - shifting	$L\{u(t - a)f(t - a)\} = e^{-as} F(s)$
6.	Differentiation of Transform	$L\{t \cdot f(t)\} = -F'(s)$
7.	Integration of Transform	$L\left\{\frac{f(t)}{t}\right\} = \int_s^\infty F(\tilde{s}) d\tilde{s}$
8.	Convolution Theorem	$L\{f(t) * g(t)\} = F(s)G(s),$ where $f(t) * g(t) = \int_0^t f(\tau)g(t - \tau) d\tau.$

Continued...

APPENDIX D:
Special Discrete Probability Distributions

Binomial Distribution, $X \sim b(x, n, p)$	
P.m.f.	$P(X = x) = {}^n C_x p^x q^{n-x}$, for $x = 0, 1, \dots, n$ and where $q = 1 - p$.
Mean	$E[X] = np$
Variance	$Var(X) = npq$
Hypergeometric Distribution, $X \sim h(x, N, n, k)$	
P.m.f.	$P(X = x) = \frac{{}^k C_x \times {}^{N-k} C_{n-x}}{{}^N C_n}$, for $x = 0, 1, \dots, \min(n, k)$.
Mean	$E[X] = \frac{nk}{N}$
Variance	$Var(X) = \frac{nk}{N} \left(\frac{N-n}{N-1} \right) \left(1 - \frac{k}{N} \right)$
Poisson Distribution, $X \sim p(x, \lambda)$	
P.m.f.	$P(X = x) = \frac{\lambda^x e^{-\lambda}}{x!}$, for $x = 0, 1, \dots$
Mean	$E[X] = \lambda$
Variance	$Var(X) = \lambda$

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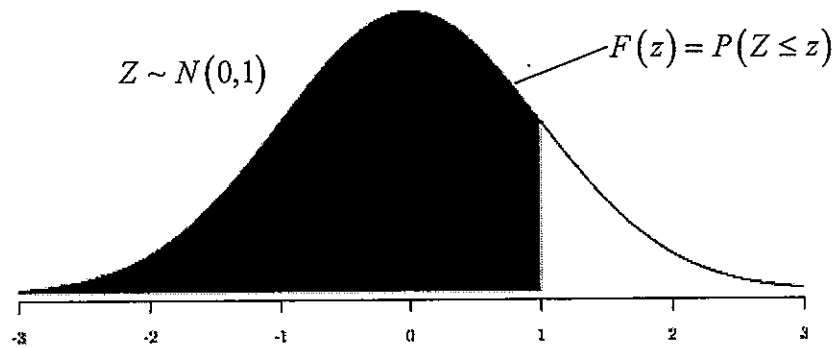
APPENDIX E:
Special Continuous Probability Distributions

Continuous Uniform Distribution, $X \sim U(a, b)$	
P.d.f.	$f_X(x) = \begin{cases} \frac{1}{b-a}, & a < x < b \\ 0, & \text{elsewhere} \end{cases}$
Mean	$E[X] = \frac{a+b}{2}$
Variance	$\text{Var}(X) = \frac{(b-a)^2}{12}$
Exponential Distribution, $X \sim \exp(1/\beta)$	
P.d.f.	$f_X(x) = \begin{cases} \frac{1}{\beta} e^{-x/\beta}, & x > 0 \\ 0, & \text{elsewhere} \end{cases}$
Mean	$E[X] = \beta$
Variance	$\text{Var}(X) = \beta^2$
Normal Distribution, $X \sim N(\mu, \sigma)$	
If X is any normal random variable where $X \sim N(\mu, \sigma)$, then the transformation $Z = \frac{X - \mu}{\sigma}$ yields a <i>standard</i> normal variable where $Z \sim N(0,1)$.	

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APPENDIX F:

Cumulative Standard Normal Distribution



	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990

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